Remarks on the UA4/2 dσ/dt

and on the Asymptotic $\sigma_T(s)$ Behavior[1]

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Abstract

We find that the UA4/2 measurements on the real part of the forward $p\bar{p}$ scattering amplitude and total cross section are consistent with each other and also with the standard picture of the Pomeron of either ln^2s or lns type. However the asymptotic $\sigma_T(s)$ behavior obtained from the *ad hoc* parametrization fit to the $p\bar{p}$ total cross sections as quoted in the Review of Particle Properties is consistent with the analysis of the class of the analytic amplitude models that contain the ln^2s type Pomeron term.

I. Preamble

Let us first recall a few definitions. The pp and $\overline{p}p$ elastic scattering amplitudes can be decomposed into their C-even and C-odd components by $F_{PP} = F_+ + F_-$ and $F_{\overline{P}P} = F_+ - F_-$ or equivalently, $F_\pm = (F_{PP} \pm F_{\overline{P}P})/2$. The total cross section is given by the optical theorem $\sigma_T(s) = (1/s)\ ImF(s, t = 0)$ and the forward ratio parameter is defined by

$$\rho(s) = \frac{ReF(s, t = 0)}{ImF(s, t = 0)} \tag{1}$$

In terms of such amplitudes, the differential cross section reads

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2(\hbar c)^2}|F(s, t)|^2 \tag{2}$$

The differential cross sections for pp and $p\overline{p}$ scattering can then be separated into Coulomb and nuclear components

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2(\hbar c)^2}|F_C + F_N|^2 \tag{3}$$

where the Coulomb part is related to the electric charge form factor by the usual formulae[1]

$$F_C = \frac{8\pi\alpha(\hbar c)^2 s}{|t|}G^2(t)\ e^{-i\alpha\phi(t)} \tag{4}$$

with the Coulomb phase approximately given by $\phi(t) = ln\,(0.08/|t|) - 0.577$ and the electric charge form factor parametrized by its dipole form $G(t) = (1 + |t|/0.71)^{-2}$. For sufficiently small angles, the hadronic amplitude obeys an exponential form

$$F_N(s,t) = F(s,t) = s\ \sigma_T(s)\ (\rho(s) + i)\ e^{-\frac{1}{2}B|t|} \tag{5}$$

Sometimes the differential cross sections are written in a simpler form by changing normalizations such that $f_C = F_C/(4\pi\hbar cs)$ etc. Given that the total cross section can be separated in elastic and inelastic counts $\sigma_T = (1/L)\ (R_e + R_i)$, $\quad d\sigma/dt = (1/L)\ (dR_e/dt)$, we obtain

$$\left(\frac{d\sigma}{dt}\right)_{t=0} = \frac{1}{16\pi s^2(\hbar c)^2}\ |F_N|^2_{t=0} = \frac{1}{16\pi s^2(\hbar c)^2}\ \sigma_T^2(1 + \rho^2) \tag{6}$$

$$\sigma_T^2(s) = \frac{16\pi(\hbar c)^2}{1 + \rho^2}\ \frac{1}{L}\ \frac{dR_e}{dt}\Big|_{t=0} \tag{7}$$

so that

$$\sigma_T(s)(1 + \rho^2) = 16\pi(\hbar c)^2\ \frac{[dR_e/dt]_{t=0}}{R_e + R_i} \tag{8}$$

which is the well-known L-independent expression. Data for $\overline{p}p$ on Eq. (8) comes both from CERN's UA4 Collaboration[2] at $\sqrt{s} = 546$ GeV, $\sigma_T(s)(1 + \rho^2(s)) = 63.3 \pm 1.5\ mb$, and from Fermilab's CDF Collaboration[3] at $\sqrt{s} = 546$ GeV, $\sigma_T(s)(1 + \rho^2(s)) = 62.64 \pm 0.95\ mb$ and at $\sqrt{s} = 1800$ GeV, $\sigma_T(s)(1 + \rho^2(s)) = 81.83 \pm 2.29\ mb$. A complete set of total cross section and real part data compilation, including statistical merging of data points at a given energy and also updated with all existing Tevatron data, can be found elsewhere[4,5].

There are two recent elastic scattering data from the UA4/2 collaboration:

(1). $\rho = 0.135 \pm 0.015$ and $B = 15.5 \pm 0.2\ (GeV/c)^{-2}$ at $\sqrt{s} = 541\ GeV$[1] in the interval $0.875 \cdot 10^{-3} \le |t| \le 0.1187\ (GeV/c)^2$ subject to the UA4 L-independent result $(1 + \rho^2)\ \sigma_T(s) = 63.3 \pm 1.5\ mb$ at $\sqrt{s} = 546\ GeV$, and

(2). The L-dependent $\sigma_T(s)$ determination[6] $\sigma_T(s) = 63.0 \pm 2.1\ mb$, which can be compared to $\sigma_T(s) = 62.2 \pm 1.5\ mb$ obtained from item (1). Similarly, from the ρ value of item (1) this total cross section gives $(1 + \rho^2)\ \sigma_T(s) = 64.15\ mb$.

II. ρ, B and σ_T from UA4/2 Experiment

We should note that the UA4/2 ρ is consistent with the standard picture of the Pomeron dominance (either $ln^2 s$ or lns type extrapolations) and thus there is little room for non-standard type of new physics[4,5]. Nevertheless, the reason behind reexaming $d\sigma/dt$ is to see if any combination of the following inputs to Coulomb fits can tolerate or even suggest alternatives to the standard picture :

(1). Dipole vs. other form factors: Felst, BSWW, etc.[7] made very little changes.

(2). Choice of t regions: But one must be careful to include enough of small t (Coulomb peak) and of large t data (to show the nuclear slope consistent with previous measurements). We selected two sets: $t = 0.875 \cdot 10^{-3}$ to $0.395 \cdot 10^{-1}$ $(GeV/c)^2$ for a total of 67 points (medium t range); and $t = 0.875 \cdot 10^{-3}$ to 0.11875 $(GeV/c)^2$ for a grand total of 99 points (full t range).

We find that these ranges affect somewhat the results, particularly the size of the parameter errors as explained below.

(3). Sensitivity of assuming that $\sigma_T(s)$ is given independently or only through the combination $(1 + \rho^2)$ σ_T: To study this, we first assume $\sigma_T(s) = 63.0 \pm 2.1 mb$ at $\sqrt{s} = 541$ GeV in the UA4/2 experiment independently of $(1 + \rho^2)$ $\sigma_T(s)$ and fit the UA4/2 t-distribution for the two t-ranges of item 2. The results of fits are as following:

medium t range :

$\sigma_T(mb)$	χ^2	$\rho(s)$	$B(GeV/c)^{-2}$
60.9	77.75	0.129 ± 0.013	15.326 ± 0.205
63.0	69.95	0.175 ± 0.014	15.084 ± 0.207
65.1	70.28	0.222 ± 0.016	14.870 ± 0.208

all t range :

$\sigma_T(mb)$	χ^2	$\rho(s)$	$B(GeV/c)^{-2}$
60.9	110.86	0.118 ± 0.008	15.546 ± 0.061
63.0	106.84	0.153 ± 0.009	15.484 ± 0.061
65.1	112.09	0.188 ± 0.010	15.428 ± 0.061

A few comments are in order about these fits:

$-$ σ_T increases as B decreases and as ρ increases indicating a negative $\rho - B$ correlation (as reported by UA4/2) ! The strengths of the correlation are, however, dependent of the t-range.

$-$ For medium t, χ^2 rises much faster at the lower end than at the higher end of $\sigma_T(s)$, while for all t, χ^2 changes slowly and symmetrically ! As expected, the smaller t-range correlates with larger parameter errors. Also for medium t, B tends to be too far from data set for higher total cross-sections (i.e., recall that $B = 15.5 \pm 0.2$ $(GeV/c)^{-2}$ for UA4/2 and $B = 15.28 \pm 0.58$ $(GeV/c)^{-2}$ for CDF). Any variation of $F_N(s,t)$ from Eq.(5) with more complicated t-dependence[7,8] must be tested with all $t - \rho$ range data to insure χ^2 stability.

$-$ For all t, $\sigma_T = 63.0$ mb and $\rho = 0.153$, which corresponds to $(1 + \rho^2)$ $\sigma_T = 64.475$ mb, i.e., within 1 σ of 63.3 ± 1.5 mb. But $\rho = 0.153 \pm 0.009$ is at 2 σ of $\rho = 0.135$ or equivalently the

experimental value $\rho = 0.135 \pm 0.015$ is at 1.2 σ of $\rho = 0.153$. Two fits of the t-distribution are shown for the two different t-ranges in Fig. (1) for comparison.

— Finally, the case in which $\rho \simeq 0.135$ and $(1 + \rho^2) \ \sigma_T = 63.3 \ mb$ (so that $\sigma_T(s) = 62.2 \ mb$) is perfectly consistent with our interpolation of ρ from the above. In fact, for all t, we get for $\sigma_T = 62.0 \ mb$, $\rho = 0.136 \pm 0.009$, $B = 15.512 \pm 0.061 \ (GeV/c)^{-2}$ and $\chi^2 = 107.45$.

(4). Sensitivity of fast slope changes: The allowed slopes from the medium and full t-range vary little, leaving little room for such variations in model fits[8]. This is clear also if one assumes $(1 + \rho^2) \ \sigma_T(s) = 63.3 \pm 1.5 \ mb$ at $\sqrt{s} = 546 \ GeV$, for then one obtains for all t-range the following fit:

$(1 + \rho^2)\sigma_T(mb)$	χ^2	$\rho(s)$	$B(GeV/c)^{-2}$	$\sigma_T(mb)$
63.3	107.25	0.137 ± 0.007	15.512 ± 0.058	62.13
64.8	107.01	0.157 ± 0.007	15.475 ± 0.058	63.23

III. Asymptotic $\sigma_T(s)$

The 1994 Review of Particle Properties[9] quotes the fits for a number of hadronic total cross sections to the *ad hoc* parametrization

$$\sigma_T = A + B \ p^n + C \ ln^2 p + D \ lnp \tag{9}$$

where $p = $ the beam momentum in GeV/c. However such a parametrization has no theoretical basis and furthemore it is difficult to give physical interpretations to the parameters, nor to provide any correlation from reaction to reaction !

On the other hand, the analytic amplitude models[4,5] based on the general principles can give natural physical interpretations to the parameters. We may then regard the analytic amplitude models as representing an "average" of the pure empirical parametrization, Eq. (9), at high energies. In particular we may extract the asymptotic $\sigma_T(s)$ behavior of Eq. (9), i.e., $\sigma_T \propto C \ ln^2 s + d \ lns + ...$ from their result and compare it with the results of the analytic amplitude model fits[5]. This can be done most appropriately for the $p\bar{p}$ reaction which has the most data at high energies. For $\bar{p}p$, the revised CERN-HERA and COMPAS fits[9] give $C = 0.26 \pm 0.05$ and $D = -1.2 \pm 0.9$.

The class of models that can be compared to their fits at high energies are either $P_2 + O + \Sigma Regges$ or $P_2 + \Sigma Regges$ types where $C = B_+$ unambiguously and $D = 2B_+(ln(2m) - lns_+) + \pi B_-$ in the notations of Ref.[5]. The models with the *lns*-type Pomeron term give $C = 0$ even though the $P_1 + R_D + R_{ND}$ model has the most preferred $\chi^2/d.o.f$ value, 1.30 for $\sqrt{s} \geq 9.7$ GeV. The results are:

$C = B_+ = 0.2425, \quad D = -0.1155 \qquad$ for the $P_2 + R_D + R_{ND}$ model ($\chi^2/d.o.f = 1.32$),

$C = B_+ = 0.2328, \quad D = -0.3273 \quad$ for the $P_2 + O + R_D + R_{ND}$ model ($\chi^2/d.o.f = 1.35$),

$C = B_+ = 0.2301, \quad D = -0.0961 \qquad$ for the $P_2 + O + R_{ND}$ model ($\chi^2/d.o.f = 1.33$, and

$C = B_+ = 0.2279, \quad D = 0.1057 \qquad$ for the $P_2 + R_{ND}$ model ($\chi^2/d.o.f = 1.37$).

IV. Conclusions

Concerning the UA4/2 ρ and σ_T, we find that:

$-$ the measurements are consistent with the standard picture of the Pomeron (either ln^2s or lns) and little room for the non-standard new physics, and

$-$ independently of assuming σ_T through $(1 + \rho^2)\sigma_T =$ fixed, one can reproduce the UA4/2 ρ, B and σ_T from their dN/dt data, i.e., ($\rho = 0.136 \pm 0.009$, $B = 15.512 \pm 0.061$ $(GeV/c)^{-2}$, $\sigma_T = 62.0$ mb) or ($\rho = 0.157 \pm 0.007$, $B = 15.475 \pm 0.058$ $(GeV/c)^{-2}$, $\sigma_T = 63.23$ mb).

Concerning the asymptotic $\sigma_T(s)$ for $p\bar{p}$, we find that:

$-$ the analytic amplitude models with ln^2s type Pomeron term give a consistent asymptotic $\sigma_T(s)$ behavior with that of the *ad hoc* parametrization fit by the CERN-HERA and COMPAS groups, i.e., $C = B_+ = 0.23 \sim 0.24$ and $D = -0.33 \sim 0.11$.

References

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Figure Captions

Fig. 1 (a). $d\sigma/dt$ for medium t and **(b)**. for all t when $\sigma_T = 63.0mb$ is assumed in the UA4/2 experiment independently of the UA4 result on $(1 + \rho^2)\sigma_T(s)$.